Section 5.2

1) $\int 2x(x^2+5)^4 dx$

$$= \int (x^2+5)^4\, 2x\,dx$$

Rewrite the problem so that the parenthesis is first:

let $u = x^2+5$

Next: $let\ u = inside\ of\ the\ parenthesis$

Rewrite the problem so that the "parenthesis is changed to an "u" $= \int u^4\, 2x\,dx$

$$u = x^2+5$$

Next find $\frac{du}{dx}$

$$\cancel{dx}\,\frac{du}{\cancel{dx}} = 2x \cdot dx$$

Multiply by dx to clear the fraction.

$$du = 2x\,dx$$

$$= \int u^4\, du$$

Next replace to make problem only have u's

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C\ \ provided\ n \neq -1$ $= \frac{1}{5}u^5 + C$

Last change $u\ back\ to\ get\ the\ answer$

$$u = x^2+5$$

$$= \frac{1}{5}(x^2+5)^5 + C$$

$answer$: $\frac{1}{5}(x^2+5)^5 + C$

3) $\int (2x+3)\,(x^2+3x-4)^3 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

$\int (x^2+3x-4)^3(2x+3)dx$

Next: $let\ u = inside\ of\ the\ parenthesis$

let $u = x^2+3x-4$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$= \int u^3\ (2x+3)dx$

Next find $\frac{du}{dx}$ $u = x^2+3x-4$

$dx\frac{du}{dx} = (2x+3)dx$

Multiply by dx to clear the fraction.

$du = (2x+3)dx$

$= \int u^3 du$

Next replace to make problem only have u's

$= \frac{1}{4}u^4 + C$

Next integrate: use Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C\ \ provided\ n \neq -1$

Last change $u\ back\ to\ get\ the\ answer$

$= \frac{1}{4}(x^2+3x-4)^4 + C$

$answer$: $\frac{1}{4}(x^2+3x-4)^4 + C$

5) $\int 2xe^{x^2}dx$

Rewrite the problem so that the “e” is written first: $= \int e^{x^2} 2x dx$

Next: $let\ u = exponent\ of\ the\ e$ $\quad let\ u = x^2$

Rewrite the problem so that the exponent is changed to an "u" $\quad \int e^{u} 2x dx$

Next find $\frac{du}{dx}$ $\quad \cancel{dx}\ \frac{du}{\cancel{dx}} = 2x\,dx$

Multiply by dx to clear the fraction. $\quad du = 2x\,dx$

Next replace to make problem only have u’s $\quad = \int e^{u} du$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$ $\quad = e^{u} + C$

Last change $u\ back\ to\ get\ the\ answer$ $\quad = e^{x^2} + C$

$answer: e^{x^2} + C$

7) $\int(2x+5)\,e^{x^2+5x}dx$

Rewrite the problem so that the "e" is written first: $= \int e^{x^2+5x}(2x+5)dx$

Next: $let\ u = exponent\ of\ the\ e$

$let\ u = x^2+5x$

Rewrite the problem so that the exponent is changed to an "u" $= \int e^{u}(2x+5)dx$

$u = x^2+5x$

Next find $\frac{du}{dx}$ $dx\frac{du}{dx} = (2x+5)dx$

Multiply by dx to clear the fraction.

$du = (2x+5)dx$

$= \int e^{u}du$

Next replace to make problem only have u's

Next integrate: "e" Rule $\int e^{x}dx \ = e^{x} + C$

$= e^{u} + C$

Last change u $back\ to\ get\ the\ answer$

$= e^{x^2+5x} + C$

$answer$: $e^{x^2+5x} + C$

9) $\int \frac{4}{4x-7} dx$

$$= \int \frac{4}{(4x-7)^1} dx$$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$$= \int 4(4x-7)^{-1} dx$$

Rewrite the problem so that the parenthesis with the exponent is first:

$$= \int (4x-7)^{-1} 4dx$$

Next: $let\ u = inside\ of\ the\ parenthesis$

$$let\ u = 4x-7$$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$$u = 4x-7 \qquad \int u^{-1} 4dx$$

Next find $\frac{du}{dx}$

$$dx \frac{du}{dx} = 4dx$$

Multiply by dx to clear the fraction.

$$du = 4dx \qquad = \int u^{-1} du$$

Next replace to make problem only have u's

$$= \ln|u| + C$$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

$$= \ln|4x-7| + C$$

Last change u $back\ to\ get\ the\ answer$

$answer$: $ln|4x-7| + C$

11) $\int \frac{2x+3}{(x^2+3x-5)} dx = \int (2x+3)(x^2+3x-5)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

Rewrite the problem so that the parenthesis with the exponent is first:

$= \int (x^2+3x-5)^{-1}(2x+3)dx$

Next: $let\ u = inside\ of\ the\ parenthesis$

let $u = x^2+3x-5$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

$u = x^2+3x-5$ $\quad = \int u^{-1}(2x+3)dx$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = (2x+3)dx$

Multiply by dx to clear the fraction.

$du = (2x+3)dx$ $\quad = \int u^{-1} du$

Next replace to make problem only have u’s

$= \ln|u| + C$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$ $\quad = \ln|x^2+3x-5| + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $ln|x^2+3x-5| + C$

#13-24: Use u-substitution to evaluate the indefinite integrals.

13) $\int 6x(x^2+5)^4 dx$ $\quad = \int (x^2+5)^4\, 6x\, dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = x^2+5$

Next: $let\ u = inside\ of\ the\ parenthesis$ $\quad = \int u^4\, 6x\, dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = x^2+5$ $\qquad = \int u^4\, 3du$

Next find $\frac{du}{dx}$

$dx\,\frac{du}{dx} = 2x\, dx$

Multiply by dx to clear the fraction.

$3\, du = 3\, 2x\, dx$ $\qquad = 3\int u^4\, du$

This is not good enough. Multiply to make a perfect match

$3du = 6x\,dx$ $\qquad = 3 \cdot \frac{1}{5} u^5 + C$

Next replace to make problem only have u's

Next integrate: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $\ provided\ n \neq -1$

$= \frac{3}{5} u^5 + C$

$= \frac{3}{5}(x^2+5)^5 + C$

Last change u $back\ to\ get\ the\ answer$

Answer: $\frac{3}{5}(x^2+5)^5 + C$

15) $\int (4x+6)\,(x^2+3x-4)^3 dx$

$$= \int (x^2+3x-4)^3 (4x+6)\,dx$$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = x^2+3x-4$

Next: $let\ u = inside\ of\ the\ parenthesis$

$$= \int u^3 (4x+6)\,dx$$

Rewrite the problem so that the “parenthesis with the exponent is changed to an "u"

$$u = x^2+3x-4$$

Next find $\frac{du}{dx}$

$$dx\frac{du}{dx} = (2x+3)\,dx$$

Multiply by dx to clear the fraction.

$$2\cdot du = 2\cdot(2x+3)\,dx$$

This is not good enough. Multiply to make a perfect match.

$$2du = (4x+6)\,dx$$

Next replace to make problem only have u’s

$$\int u^3 \cdot 2\,du$$

Next integrate: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C$ $\ provided\ \overset{n}{\cancel{x}} \neq -1$

$$= 2\int u^3 du$$

$$= \overset{1}{\cancel{2}} \cdot \frac{1}{\underset{2}{\cancel{4}}}\, u^4 + C$$

Last change $u\ back\ to\ get\ the\ answer$

$$= \frac{1}{2}u^4 + C$$

Answer: $\frac{1}{2}(x^2+3x-4)^4 + C$

$$= \frac{1}{2}(x^2+3x-4)^4 + C$$

17) $\int 10xe^{x^2}dx$ $= \int e^{x^2} 10x dx$

Rewrite the problem so that the "e" is written first:

let $u = x^2$

Next: $let\ u = exponent\ of\ the\ e$

$= \int e^u 10x dx$

Rewrite the problem so that the exponent is changed to an "u"

$u = x^2$

Next find $\frac{du}{dx}$

$dx \frac{du}{dx} = 2x dx$

Multiply by dx to clear the fraction.

$5 \cdot du = 5 \cdot 2x dx$

This is not good enough. Multiply to make a perfect match.

$10 du = 10x dx$

$\int e^u 10 du$

Next replace to make problem only have u's

$= 10 \int e^u du$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

$= 10e^u + C$

Last change $u\ back\ to\ get\ the\ answer$

$= 10e^{x^2} + C$

$answer$: $5e^{x^3} + C$

19) $\int (8x + 20)\, e^{x^2+5x} dx$

$$= \int e^{x^2+5x}(8x+20)dx$$

Rewrite the problem so that the “e” is written first:

$$\text{let } u = x^2+5$$

Next: $let\ u = exponent\ of\ the\ e$

$$= \int e^{u}(8x+20)dx$$

Rewrite the problem so that the exponent is changed to an "u"

$$u = x^2+5x$$

Next find $\frac{du}{dx}$

$$dx\frac{du}{dx} = 2x+5dx$$

Multiply by dx to clear the fraction.

$$4\,du = 4(2x+5)dx$$

This is not good enough. Multiply to make a perfect match.

$$4du = (8x+20)dx$$

Next replace to make problem only have u’s

$$= \int e^{u} \cdot 4du$$

Next integrate: "e" Rule $\int e^x dx \ = e^x + C$

$$= 4\int e^{u} du$$

Last change u $back\ to\ get\ the\ answer$

$$= 4e^{u} + C$$

$answer$: $4e^{x^2+5x} + C$

$$= 4e^{x^2+5x} + C$$

21) $\int \frac{8}{4x-7} dx$ $= \int 8(4x-7)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (4x-7)^{-1} 8 dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 4x-7$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^{-1} 8 dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 4x-7$

Next find $\frac{du}{dx}$ $dx \frac{du}{dx} = 4 dx$

Multiply by dx to clear the fraction. $2du = 2 \cdot 4dx$

$2du = 8dx$

This is not good enough. Multiply to make a perfect match.

$= \int u^{-1} \cdot 2du$

Next replace to make problem only have u's

$= 2\int u^{-1} du$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

$= 2\ln|u| + C$

$= 2\ln|4x-7| + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $2ln|4x - 7| + C$

23) $\int \frac{16x+24}{x^2+3x-5}$ $= \int (16x+24)(x^2+3x-5)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (x^2+3x-5)^{-1}(16x+24) dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = x^2+3x-5$

Next: $let\ u = inside\ of\ the\ parenthesis$

$\int u^{-1}(16x+24) dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = x^2+3x-5$

Next find $\frac{du}{dx}$

$dx \frac{du}{dx} = (2x+3) dx$

Multiply by dx to clear the fraction.

$8\ du = 8(2x+3) dx$

This is not good enough. Multiply to make a perfect match.

$8du = (16x+24) dx$

$= \int u^{-1} \cdot 8du$

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1} dx = \ln|x| + C \\ \int \frac{1}{x} dx = \ln|x| + C \end{cases}$

$= 8 \int u^{-1} du$

$= 8 \ln|u| + C$

Last change u $back\ to\ get\ the\ answer$

$answer$: $8ln|x^2+3x-5|+C$

$= 8 \ln|x^2+3x-5| + C$

#25-32: Use u-substitution to evaluate the indefinite integrals.

25) $\int 2x(4x^2+5)^4dx$ $= \int (4x^2+5)^4\, 2x \cdot dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 4x^2+5$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^4 \cdot 2x\,dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 4x^2+5$

Next find $\frac{du}{dx}$

$dx\,\frac{du}{dx} = 8x\,dx$

Multiply by dx to clear the fraction.

$\frac{1}{4}du = \frac{1}{4}8x\,dx$

This is not good enough. Multiply to make a perfect match

$\frac{1}{4}du = 2x\,dx$

Next replace to make problem only have u's

$\int u^4 \cdot \frac{1}{4}du$

Next integrate: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ \cancel{x}\ n \neq -1$

$= \frac{1}{4}\int u^4 du$

$= \frac{1}{4} \cdot \frac{1}{5}u^5 + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\frac{1}{20}(4x^2+5)^5 + C$

$= \frac{1}{20}u^5 + C$

$= \frac{1}{20}(4x^2+5)^5 + C$

27) $\int(2x+3)\,(2x^2+6x-1)^3dx$

$$= \int (2x^2+6x-1)^3(2x+3)\,dx$$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 2x^2+6x-1$

Next: $let\ u = inside\ of\ the\ parenthesis$

$$= \int u^3(2x+3)\,dx$$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 2x^2+6x-1$

Next find $\frac{du}{dx}$

$$dx\,\frac{du}{dx} = (4x+6)\,dx$$

Multiply by dx to clear the fraction.

$$\frac{1}{2}du = \frac{1}{2}(4x+6)\,dx$$

This is not good enough. Multiply to make a perfect match

$$\frac{1}{2}du = (2x+3)\,dx$$

Next replace to make problem only have u's

$$= \int u^3 \cdot \frac{1}{2}du$$

Next integrate: Power Rule: $\int x^n dx = \frac{1}{n+1}x^{n+1} + C \quad provided\ \cancel{x}\ n \neq -1$

$$= \frac{1}{2}\int u^3du$$

Last change $u\ back\ to\ get\ the\ answer$

$$= \frac{1}{2}\cdot\frac{1}{4}u^4 + C$$

$answer$: $\frac{1}{8}(2x^2+6x-1)^4 + C$

$$= \frac{1}{8}u^4 + C$$

$$= \frac{1}{8}(2x^2+6x-1) + C$$

29) $\int \frac{2}{4x-7}dx$ $= \int 2(4x-7)^{-1}dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (4x-7)^{-1}2dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 4x-7$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^{-1}2dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 4x-7$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = 4dx$

Multiply by dx to clear the fraction.

$\frac{1}{2}du = \frac{1}{2}4dx$

This is not good enough. Multiply to make a perfect match.

$\frac{1}{2}du = 2dx$

$= \int u^{-1}\cdot\frac{1}{2}du$

Next replace to make problem only have u's

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

$= \frac{1}{2}\int u^{-1}du$

$= \frac{1}{2}\ln|u| + C$

Last change $u\ back\ to\ get\ the\ answer$

$answer$: $\frac{1}{2}ln|4x-7| + C$

$= \frac{1}{2}\ln|4x-7| + C$

31) $\int \frac{2x+2}{3x^2+6x-5} dx = \int (2x+2)(3x^2+6x-5)^{-1} dx$

Rewrite so that the problem is not a fraction, and has a parenthesis with a -1 exponent

$= \int (3x^2+6x-5)^{-1}(2x+2)\,dx$

Rewrite the problem so that the parenthesis with the exponent is first:

let $u = 3x^2+6x-5$

Next: $let\ u = inside\ of\ the\ parenthesis$

$= \int u^{-1}(2x+2)\,dx$

Rewrite the problem so that the "parenthesis with the exponent is changed to an "u"

$u = 3x^2+6x-5$

Next find $\frac{du}{dx}$

$dx\frac{du}{dx} = (6x+6)dx$

Multiply by dx to clear the fraction.

$\frac{1}{3}du = \frac{1}{3}(6x+6)dx$

This is not good enough. Multiply to make a perfect match.

$\frac{1}{3}du = (2x+2)dx$

Next replace to make problem only have u's

$= \int u^{-1}\frac{1}{3}du$

Next integrate: "ln" Rule: $\begin{cases} \int x^{-1}dx = \ln|x| + C \\ \int \frac{1}{x}dx = \ln|x| + C \end{cases}$

$= \frac{1}{3}\int u^{-1}du$

Last change $u\ back\ to\ get\ the\ answer$

$= \frac{1}{3}\ln|u| + C$

$answer$: $\frac{1}{3}ln|3x^2+6x-5| + C$

$= \frac{1}{3}\ln|3x^2+6x-5| + C$